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Properties of a weakly interacting electronic system under a staggered magnetic field

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Abstract. We study the properties of a two-dimensional interacting electronic system in the presence of a staggered magnetic field. We find that metal–insulator transitions can be achieved at both zero and non-zero temperatures. A d-wave-like pseudogap is opened on the two-dimensional square Fermi surface. The staggered magnetic flux is found to suppress the antiferromagnetic order. When random fluctuations of the perfect staggered field are turned on and increased, a finite region of Fermi surface around $(\pm\pi/2, \pm\pi/2)$ will be gradually formed as the bandwidth decreases. We also find that the assumed disturbed staggered field in our model can actually simulate the spin fluctuations occurring in the interacting system.

1. Introduction

Since the discovery of the quantum Hall effect [1, 2] and colossal magnetoresistance [3, 4], quantum systems under external fields have attracted much attention. Among their many characteristic properties, the field-induced phase transition is particularly interesting to both theoreticians and experimentalists. Theoretically, the magnetic-field-induced metal–insulator transition due to a random magnetic field in two dimensions has been studied by several authors [5–7]. They have argued that in the presence of a random magnetic field the states in the centre of the band are delocalized and the density of states there has a singularity. Moreover, Ryuichi Ugajin also found a sharp *electric*-field-induced metal–insulator Mott transition in a two-dimensional two-layered system [8, 9].

More recently, the electronic transmission properties for inhomogeneous flux in one-dimensional chains have been studied numerically; very interesting transmission behaviour has been found, which might be useful in applications in the fabrication of special devices [10–12]. In fact, it is now experimentally possible to construct and detect a field that is inhomogeneous at scales well below micrometres, and thus quantum system properties under magnetic fields are also very attractive and important to experimentalists.

One purpose of this paper is to study from a different viewpoint the properties of a two-dimensional interacting electronic system in the presence of a staggered magnetic field whose quantized staggered fluxes go through the plaquettes of the lattice alternately. The model employed here is the Hubbard model, which is defined by

$$H = -t \sum_{\langle ij\rangle\sigma} c^{+}_{j\sigma} c_{i\sigma} + U \sum_{i} n_{i\sigma} n_{i-\sigma} \tag{1}$$

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where $\langle ij \rangle$ denote nearest neighbours, t hopping transfers, and U the Coulomb repulsion which accounts for electron–electron correlations. For exactly one electron per site, i.e., a half-filled band, it has become fairly well known that this model for large U has a two-sublattice (commensurate) antiferromagnetic (AF) insulating ground state with all electrons localized. However, for smaller U, although the electrons are itinerant rather than localized, the ground state is still the antiferromagnetic one because of the instability due to the nested Fermi surface.

Another purpose of this paper is to investigate whether the assumed staggered magnetic field can account for the antiferromagnetic fluctuations occurring in the interacting system. Generally, one may introduce a staggered magnetic field into the system by carrying out a local gauge transformation of the electron's internal coordinate, in which the interaction term is invariant and which may cause electrons circling any plaquette of a 2D square lattice to suffer a phase shift π [13]. The state is known as the π-phase flux state. This is a direct consequence of the two-valuedness of spin-1/2 wave functions and the doubly connected topology of the rotation group manifold SO(3) [14, 15]. Therefore, a staggered magnetic flux with half of the flux quanta can be introduced so as to go through each plaquette identically. Furthermore, a random magnetic field can also be introduced if a random gauge field is considered [1, 16, 17]. Moreover, in references [18, 20], a rather general flux state (a θ-phase flux state, $\theta \neq \pi$) is introduced in the t–J model by the authors using a Hubbard–Stratonovich transformation to compete with the d-wave-pairing state.

However, on the other hand, the disturbed staggered magnetic field can actually be seen to come from the antiferromagnetic fluctuations. The reason for this is the following: the effective flux through each plaquette ϕ_i which acts on electrons can be seen to be proportional to the sum of the spin fluctuations on the four corners of the plaquette i. Thus correlations between these fluxes can be attributed to the correlations between spin fluctuations $\langle \delta \boldsymbol{S}_i \cdot \delta \boldsymbol{S}_j \rangle$, where

$$\boldsymbol{S}_i = \sum_{\alpha\beta} \boldsymbol{\sigma}_{\alpha\beta} c_{i\alpha}^+ c_{i\beta}$$

is the local spin operator at the site i. Although the flux averages to zero, it can actually deviate from zero due to spin fluctuations, and then cause the electrons to feel an effective fluctuating magnetic field with the quantized ϕ averaging to zero. Furthermore, the field is a fluctuating staggered field, because antiferromagnetic correlations would cause the neighbouring fluxes to correlate with each other oppositely: $\langle \phi_i \phi_j \rangle = -f(x, T)$, where f is a positive quantity which can be dependent on the temperature T or hole doping x. The concrete form of f will be determined elsewhere [19]. We can treat this flux fluctuation approximately by assuming that $\phi_i = (-1)^i \phi + \delta\phi_i$, where $\phi = \sqrt{f}$ and the $\delta\phi_i$ are random fluctuations which are such that $\langle \delta\phi_i \rangle = 0$ and $\langle \delta\phi_i \, \delta\phi_j \rangle = 0$. Thus a disturbed staggered flux is introduced. The reason that we can proceed in this way is that this staggered flux state is actually more energetically favourable than all the other states with non-zero flux (see the appendix). This is physically because the staggered pattern, which does not break the translational symmetry, has a flux equivalent to zero for any magnetic unit cell of the lattice [20].

Note that although the correlations between the fluxes are short range, we would like to consider here an ideal staggered flux $\{\phi, -\phi\}$ to account for them. The random fluctuations $\delta\phi$ can be seen to come physically from the long-range disorder. Since the spin fluctuation is sensitive to temperature and hole doping, the quantum fluxes here are also actually temperature and doping dependent. In fact, in the model employed here, increasing spin fluctuations would correspond to a process of weakening of the short-range correlation (i.e., decreasing ϕ) and, simultaneously, of strengthening of the long-range disorder (i.e., increasing the amplitude of $\delta\phi$).

The paper is organized as follows. In the next section, we study the effects of a perfect staggered flux. We give the electron density of states (DOS) to show the suppression of the long-range order due to the staggered field. The Fermi surface at half-filling is then found to be point-like, giving a d-wave pseudogap and power laws. The possibility of metal–insulator transitions with varying ϕ is also argued for. The induced current is finally given, and shows very interesting temperature- and flux-dependent behaviour. In section 3, the case of the disturbed staggered flux is discussed. The evolution of the Fermi surface with the random fluctuations is given. We find that this is very similar to what has been observed in angle-resolved photoemission spectra (ARPES) experiments for underdoped cuprates [21]. In section 4, we summarize our results and conclusions.

2. Features under a perfect staggered magnetic field

Now we discuss our model. We restrict ourselves to the half-filled case. In this particular situation, there exists long-range antiferromagnetic ordering for any non-zero U. Although the Hartree–Fock (HF) approximation usually overestimates the symmetry breaking, it does give the correct answer here (in the weak-coupling limit), as validated by early analytical and numerical studies. After the HF decoupling process, the mean-field Hamiltonian can be given by (the constant term is omitted)

$$H_{MF} = -t \sum_{\langle ij \rangle \sigma} (c_{i\sigma}^{+} c_{j\sigma} + \text{h.c.}) - mU \sum_{k\sigma} \sigma c_{k+Q\sigma}^{+} c_{k\sigma} \tag{2}$$

where $\boldsymbol{Q} = (\pi, \pi)$, and m is the antiferromagnetic order parameter, defined by

$$m = \frac{1}{N} \sum_{i} \langle (n_{i\uparrow} - n_{i\downarrow}) \rangle.$$

With staggered magnetic fluxes introduced to go through the plaquettes of the square lattice, as shown in figure 1, an interesting mechanism of competition between the Coulomb correlation and the staggered field would appear. The Coulomb repulsive correlation U tends to arrange the nearest-neighbour spins antiferromagnetically, i.e., to maintain an antiferromagnetic order (or spin-density-wave (SDW) order). On the other hand, the staggered

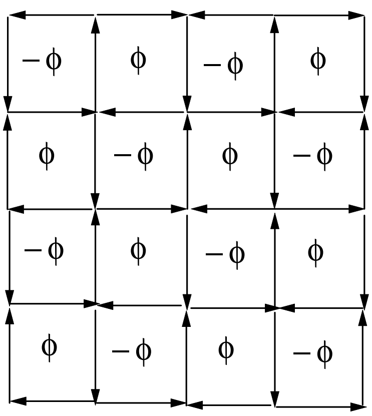

Figure 1. A two-dimensional square lattice, through each square plaquette of which the fluxes ϕ and $-\phi$ are threaded alternately. Any electrons hopping along the directions indicated by the arrows in the figure would suffer a phase shift of $\delta/4$, where $\delta = 2\pi\phi/\phi_0$.

magnetic flux in each plaquette tends to turn the spins on the corners of each plaquette to make them run parallel in the direction of the flux, i.e., it will suppress the antiferromagnetic order. Thus a larger critical Coulomb correlation U_c is needed to switch on this long-range ordering.

This can also be easily understood if we see the staggered field as a simulation of spin fluctuations, which become large when, for example, a doped hole is introduced, and then cause a non-zero critical U_c to stabilize a long-range order. As is known, the HF approximation always overestimates the electron correlations because of its mean-field nature. Thus in this situation, it is more reasonable and physical to consider a staggered flux to account for the fluctuations, which would reduce the overestimation of the correlations. Thus from this viewpoint, this method has gone beyond the conventional mean-field method for the interacting system without an external field.

The hopping transfers of electrons, t_{ij}, should be modified, because electrons hopping from one site to its neighbours would suffer phase shifts, according to the AB effect [22], which is a special manifestation of Berry's phase [23]. The selection of the distribution of these phase shifts is arbitrary as long as it ensures that an electron circling a plaquette once suffers a phase shift of $\delta = 2\pi\phi/\phi_0$, which is gauge invariant. Here, we adopt a symmetric phase configuration, as shown in figure 1. An electron hopping along (against) the direction of the arrows suffers a phase shift of $\delta/4$ $(-\delta/4)$. Specifically, one has $t_{ij} = t\mathrm{e}^{\mathrm{i}\delta_{ij}}$ and

$$\langle ij\rangle = (0, 1), (0, -1), (1, 0), (-1, 0)$$
$$\delta_{ij} = -\delta/4, -\delta/4, \delta/4, \delta/4.$$

Thus the mean-field Hamiltonian in k-space reads

$$H_{MF} = \sum_{k\sigma}\{\cos(\delta/4)\epsilon_k c_k^+ c_k - (Um\sigma + \mathrm{i}\sin(\delta/4)\Delta_k)c_{k+Q\sigma}^+ c_{k\sigma}\} \tag{3}$$

where $\epsilon_k = -2t(\cos k_x + \cos k_y)$ is the band energy and $\Delta_k = 2t(\cos k_x - \cos k_y)$ is a quantity with d-wave symmetry; so the energy spectrum can be given by

$$\omega_k^{\pm} = \pm\sqrt{\cos^2(\delta/4)\epsilon_k^2 + \sin^2(\delta/4)\Delta_k^2 + (Um)^2} \tag{4}$$

where the antiferromagnetic order parameter m is determined by the following self-consistent equation:

$$1 = \frac{U}{2N}\sum_k \frac{f(\omega_k^-) - f(\omega_k^+)}{\sqrt{\cos^2(\delta/4)\epsilon_k^2 + \sin^2(\delta/4)\Delta_k^2 + (Um)^2}}. \tag{5}$$

At half-filling, the lower sub-band is filled completely, with the upper sub-band empty. In the absence of the magnetic field, the Fermi surface is nested by $\boldsymbol{Q} = (\pi, \pi)$. However, as one turns on the staggered magnetic flux, the Fermi surface becomes four points: $(\pm\pi/2, \pm\pi/2)$. Note that this point-like Fermi surface is a general feature of the flux phase [13]. The original square Fermi surface was concave inside, as illustrated in figure 2 by the shifting of the peaks of the density of states (DOS) as a function of $p = \phi/\phi_0$ for $m = 0$. In reference [15], John and Müller-Groeling give a similar DOS for the π-phase flux state in the absence of SDW order. Because of this property of the DOS, the integral or sum in equation (5) would always converge, which is quite different from the situation where the magnetic field is absent. Thus at zero temperature there exists a non-zero critical value U_c for the onset of antiferromagnetic order:

$$U_c = \left(\frac{1}{2N}\sum_k |\cos^2(\delta/4)\epsilon_k^2 + \sin^2(\delta/4)\Delta_k^2|^{-1/2}\right)^{-1}. \tag{6}$$

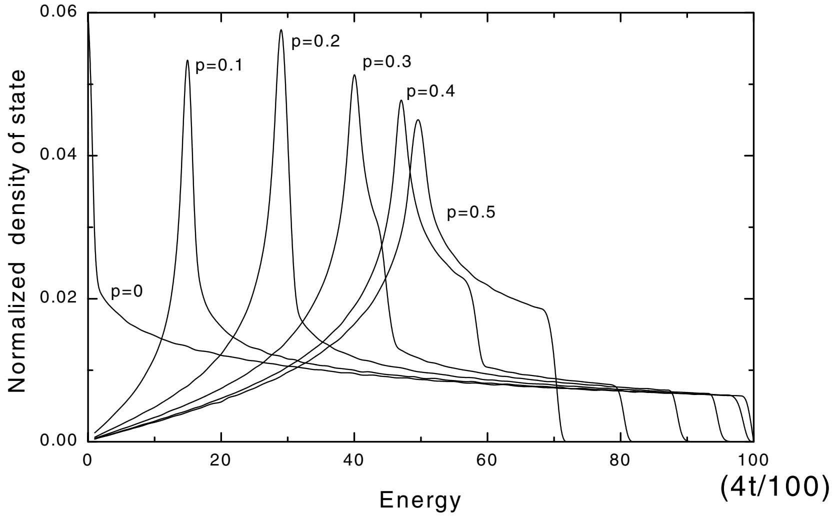

Figure 2. Normalized density of states versus energy for different magnetic fields, where $p = \phi/\phi_0$ and $m = 0$. The curves are symmetric with respect to $p = 0.5$, i.e., the curve for a value of p which is less than 0.5 is identical to that for $1 - p$.

We show the dependence of U_c on magnetic flux ϕ in figure 3. Note that in the absence of the staggered field any small U would stabilize an antiferromagnetic order at half-filling. This unusual non-zero U_c can also be produced if we introduce into the model the next-nearest-neighbour hoppings to disturb the antiferromagnetic order instead of a magnetic field [24,25]—although the mechanisms are different. Here, the non-zero U_c is caused by the magnetic flux and it can be changed as one tunes the flux. Thus we have just proved the statement made

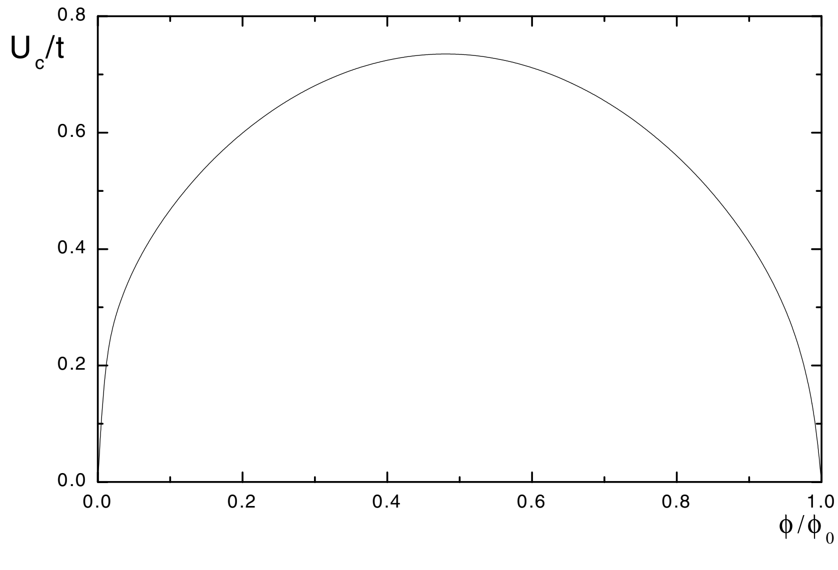

Figure 3. The critical value U_c/t versus the magnetic flux ratio ϕ/ϕ_0. The curve is symmetric with respect to $\phi = 0.5\phi_0$, and periodic with period ϕ_0.

earlier that the existence of staggered magnetic flux suppresses the antiferromagnetic ordering. In fact, in the large-U limit, we also have

$$m = \frac{1}{2} - 4\frac{t^2}{U^2} + 3(1+\cos^2\delta)\frac{t^4}{U^4} \tag{7}$$

and it can be easily seen that having a finite δ decreases m.

Now we can achieve a magnetic-field-induced metal–insulator transition when modulating ϕ if U is so small that it is actually less than the maximum value of U_c, i.e., $U < U_c^{\max}$. This transition happens because the long-range order would disappear in a certain region of ϕ (where $U_c(\phi) > U$) where the system has been changed from an insulator (where $U_c(\phi) < U$) with a finite gap into a gapless metal. Since the staggered flux suppresses the long-range order, the Néel temperature in the presence of a magnetic field, $T_c(\phi)$, would be smaller than that in the absence of a magnetic field, $T_c(0)$. At finite temperatures (with $T_c(\phi_0/2) < T < T_c(0)$), however, one can expect another type of field-induced metal–insulator transition to occur only if ϕ is modulated continuously in a period.

This existence of two types of field-induced transition can actually be understood if we approach our consideration from the viewpoint that the staggered field is a simulation of the spin fluctuations—the transitions just arise from the equivalent fact that when, for example, holes are doped in, which causes the fluctuations to become stronger, the long-range antiferromagnetic order will disappear.

In the absence of long-range order, the bandwidth W can be given by

$$W = 4t\cos(\delta/4) \qquad (|\phi| < \phi_0/2). \tag{8}$$

From figure 2, it can be seen that the bandwidth is a decreasing function of ϕ with ϕ ranging from 0 to $(1/2)\phi_0$. This means that the electron effective hopping would be suppressed by increasing the magnetic flux within that region. Moreover, an anisotropic 'd-wave'-type pseudogap would be opened on the original Fermi surface, where $\epsilon_k = 0$:

$$\widetilde{\Delta_k} = 4t|\sin(\delta/4)||\cos k_x - \cos k_y| \qquad (|\phi| < \phi_0/2). \tag{9}$$

Because of this pseudogap, most thermodynamical properties of the system should follow a power law at low temperatures. In order to illustrate the modulation of the power law by the magnetic field, we give below the density of states near the Fermi surface explicitly:

$$\begin{aligned} &\rho(\varepsilon) = \frac{\pi}{|\sin(\delta/2)|t^2}|\varepsilon| \qquad \text{when } |\varepsilon| \ll t|\sin\delta/2| \\ &(\rho_N(\varepsilon) \propto \ln|\varepsilon| \qquad \text{when } |\varepsilon| \ll t|) \end{aligned} \tag{10}$$

where $\rho_N(\varepsilon)$ is the density of states for the normal case with $\phi = 0$. Note that this linear behaviour near zero energy has already been shown in figure 1. Thus the temperature dependence of the specific heat and the rate of absorption modulated by the magnetic flux will then obey the power laws

$$\begin{aligned} &C \propto \frac{T^2}{|\sin(\delta/2)|} \qquad (C_N \propto T) \qquad \text{when } k_B T \ll t|\sin(\delta/2)| \\ &\frac{\mathrm{Re}\,\sigma}{\mathrm{Re}\,\sigma_N} \propto \begin{cases} \dfrac{\omega}{|\sin(\delta/2)|} & \text{when } k_B T \ll \omega \ll t|\sin(\delta/2)| \\ \dfrac{T}{|\sin(\delta/2)|} & \text{when } \omega \ll k_B T \ll t|\sin(\delta/2)| \end{cases} \end{aligned} \tag{11}$$

where C_N and $\mathrm{Re}\,\sigma_N$ are the quantities corresponding to the case with $\phi = 0$. As expected, the system behaves like something between a metal and an insulator, i.e., a semi-metal.

Before ending this section, we would like to give the field-induced current and its distribution. To be explicit, let us consider the current flowing from one site to its nearest neighbour in the x-direction [26]:

$$I_x = \frac{2et}{\hbar}\,\mathrm{Im}(\exp(\mathrm{i}\delta/4)\langle c^+_{i+\hat{x}}c_i\rangle) = \frac{4et}{h}\sin(\delta/2)A(\delta,T) \tag{12}$$

where

$$A(\delta,T) = \frac{a^2}{2\pi}\int_{k\in\mathrm{BZ}} \mathrm{d}k\,\cos(k_xa)\cos(k_ya) \times \frac{\tanh\left((\beta/2)\sqrt{\cos^2(\delta/4)\epsilon^2_{k} + \sin^2(\delta/4)\Delta^2_{k} + (Um)^2}\right)}{\sqrt{\cos^2(\delta/4)\epsilon^2_{k} + \sin^2(\delta/4)\Delta^2_{k} + (Um)^2}}. \tag{13}$$

The flux dependences of the current at different temperatures are shown in figure 4. The currents are functions of ϕ with period ϕ_0 and decrease with temperature. They change smoothly through zero. In a similar way, one can obtain the current flowing in the other directions: $I_{-x} = I_x = -I_y = -I_{-y}$. Using the translational symmetry, one can get the distribution of current for the whole system. The distribution is exactly the same as that shown by the arrows in figure 1. The only difference is that in a period ϕ_0, the direction of the current changes twice.

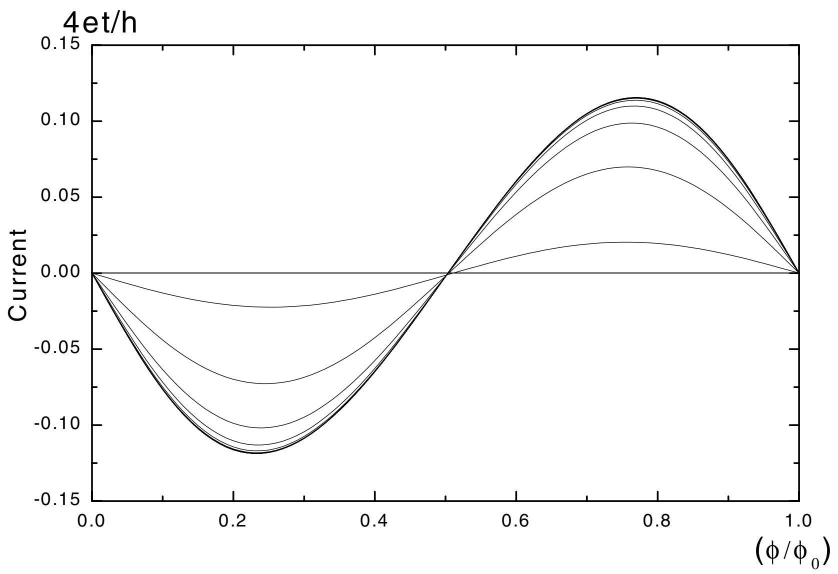

Figure 4. Current versus magnetic flux ratio ϕ/ϕ_0 for various temperatures, where the order parameter $\Delta = Um/t$ is set to be 1. The temperatures are $kT = 2t, 2t/2, 2t/3, \ldots 2t/6$. The current is gradually suppressed with increasing temperature.

3. Features under a disturbed staggered magnetic field

In section 2, we have studied the properties of the system under a perfect staggered magnetic field. However, from the viewpoint of the simulation of the spin fluctuations, the staggered flux is a special pattern of the fluctuating fluxes. The reason that we can proceed in this way is that, as mentioned before, the staggered pattern is the most energetically favourable among the fluctuating ones. In the appendix, we have considered a series of translational-symmetry-broken flux patterns which deviate from the staggered one in a perturbational way, and have shown that the energy of the staggered flux state is actually the minimum one.

Since the staggered flux state is the most favourable one, it is reasonable to expect the staggered field to account for the short-range antiferromagnetic order. However, the long-range disorder has not been considered until now, and in actual physical situations, it is so important that it cannot be neglected. In order to demonstrate its importance, in this section we would like to describe the evolution of the Fermi surface with the disturbed flux. Here, we restrict ourselves to the half-filling case, i.e., there is no hole doping, so from the viewpoint of simulation of the spin fluctuations, we would like to give equivalently the evolution of the Fermi surface with temperature, not hole doping. We still assume $m = 0$, which means that the long-range antiferromagnetic order is absent. To start, we discuss the case corresponding to the magnetic flux configuration as shown in figure 5(b) with $\phi_1^\pm = \phi \pm \delta\phi_1, \phi_2^\pm = \phi \pm \delta\phi_2$, where $\delta\phi_{1\,(2)}$ are small deviations. For this special deviated-flux configuration, we have calculated the energy dispersion and found that the points with zero gaps occur at the following 16 points:

$$\left(\pm\frac{\pi}{2} \pm w(-), \pm\frac{\pi}{2} \pm w(+)\right) \tag{14}$$

where

$$w(\pm) = \frac{\pi}{2\phi_0}(\delta\phi_1 \pm \delta\phi_2). \tag{15}$$

Thus, compared with the case of perfect staggered flux, equation (9), the number of points with zero gap is quadrupled. For the configuration shown in figure 5(a), which is a special case of figure 5(b) with $\delta\phi_1 = \delta\phi_2$, the number of points with zero gap is just doubled. Note that the position of these points is just the Fermi surface of the system at half-filling, which is point-like as before, but with the number of points increased. So we expect to obtain a finite

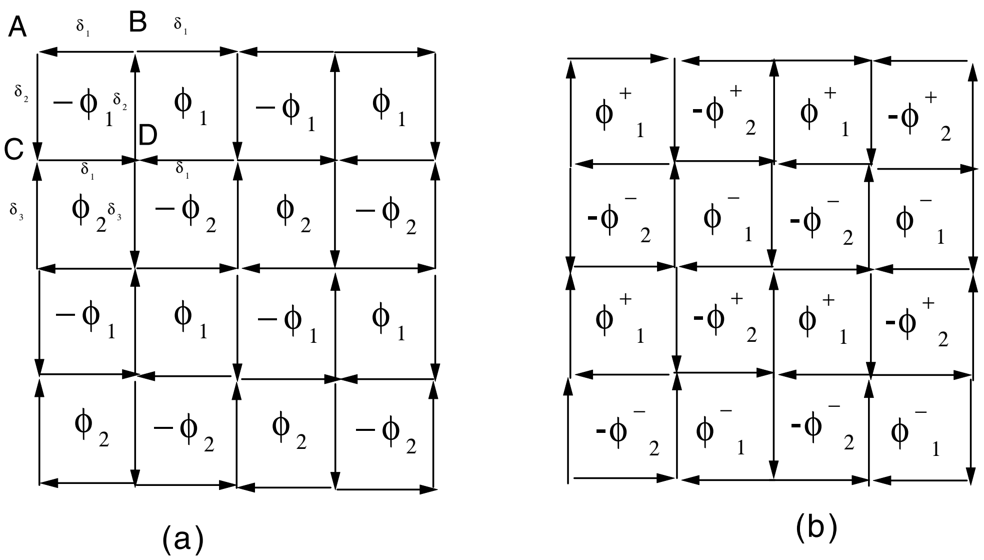

Figure 5. Two symmetric flux deviations from the staggered magnetic field. A, B, C, D are the four types of site in the corresponding sublattices, and δ_1, δ_2 are the phase shifts of electrons hopping along the directions indicated by the arrows. (a) is a special case of (b) with $\delta\phi_1 = \delta\phi_2$.

density of states with zero gap on the Fermi surface as the fluctuation in the magnetic flux becomes strong.

To investigate this problem, we introduce random fluctuations, which should account for the long-range disorder, into the staggered magnetic flux ϕ such that

$$\phi_i = (-1)^i \phi + \delta\phi \,\mathrm{RAND}[-1, 1] \tag{16}$$

where RAND$[-1, 1]$ stands for random numbers uniformly distributed from -1 to 1 and $\delta\phi$ is the amplitude of the random fluctuations. For random ϕ_i, analytical solution is impossible, so we diagonalize the Hamiltonian matrix numerically for a 40×40 site lattice as an illustration. In figure 6, we present results for the DOS for $\phi = 0.2$, $\delta\phi = 0.2, 0.4, 0.6, 0.8, 1.0$, and 1.2 (in units of ϕ_0). We observe that as $\delta\phi$ is increased:

(1) the system has more and more states with zero gap, i.e., the Fermi surface has evolved from several points to a finite region around points $(\pm\pi/2, \pm\pi/2)$, which is consistent with the conclusion that we drew from figure 5(b). Since the states at the centre of the energy band are delocalized, i.e., extended [5, 6], the conductivity of the half-filled band would increase continuously with fluctuation of the staggered magnetic field. Note that this behaviour of the evolution of the pseudogap with the fluctuation of the staggered magnetic field is very similar to what is observed for underdoped cuprates in ARPES experiments [21]. This can be understood well if we see it as the evolution of the Fermi surface with temperature, on the basis of the viewpoint that the disturbed staggered flux is a simulation of the spin fluctuations which can be temperature dependent. Note also that in reference [27], a fluctuating U(1) staggered gauge field is studied, to obtain the evolution of the Fermi surface with hole doping in underdoped cuprate. Moreover, the dependence of the DOS there on the temperature and doping is quite similar to that obtained here on the random fluctuating flux. We do not know whether the disturbed staggered flux and the fluctuating staggered gauge field are the same thing, but we believe that they should be correlated in some way.

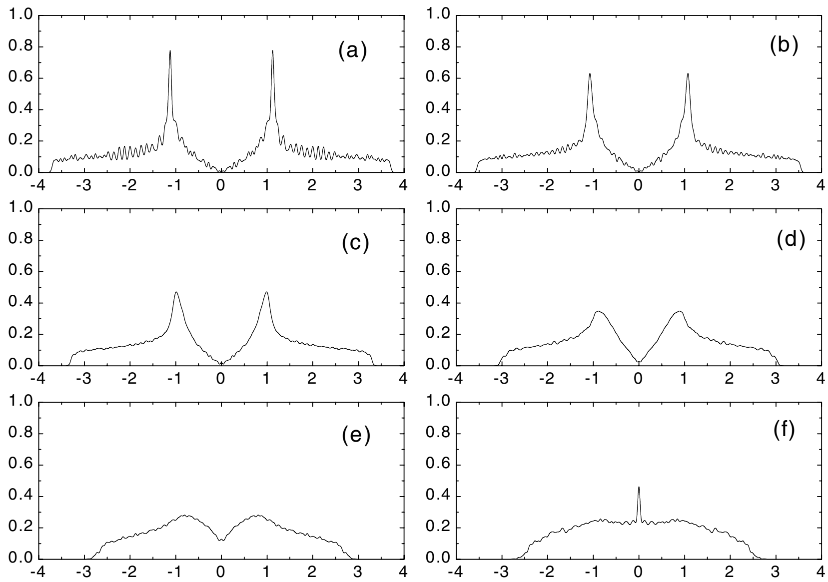

Figure 6. The density of states for the disturbed staggered magnetic field with $\phi = 0.2\phi_0$ and (a) $\delta\phi = 0.2\phi_0$, (b) $\delta\phi = 0.4\phi_0$, (c) $\delta\phi = 0.6\phi_0$, (d) $\delta\phi = 0.8\phi_0$, (e) $\delta\phi = 1.0\phi_0$, (f) $\delta\phi = 1.2\phi_0$.

(2) When $\delta\phi$ is large enough, the DOS resembles that for the tight-binding model in the absence of the field, because its singularity at zero energy has been recovered gradually. The completely random cases for $\phi = 0$ have also been calculated and the singularities at zero energy are still present (see figure 7). This feature is in agreement with the previous result calculated by other authors [5, 28]. Note that the case for $\phi = 0$ can be seen from the viewpoint of the simulation of fluctuations as a situation where the fluctuation is so large that even the short-range order has already disappeared.

(3) The bandwidth is a decreasing function of $\delta\phi$. This feature does not change for the case where $\phi = 0$. This means that the effective electron hopping can be suppressed by the fluctuation of the staggered magnetic field. Some wiggles appearing in figure 5 are finite-size effects.

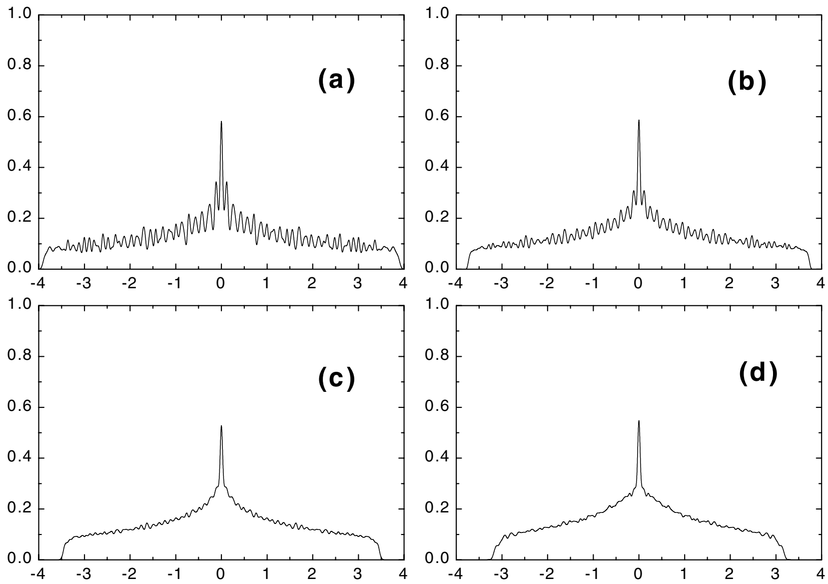

Figure 7. The density of states for the random magnetic field with $\phi = 0$ and (a) $\delta\phi = 0.2\phi_0$, (b) $\delta\phi = 0.4\phi_0$, (c) $\delta\phi = 0.6\phi_0$, (d) $\delta\phi = 0.8\phi_0$.

4. Conclusions

In conclusion, we have studied properties for an interacting system under a staggered magnetic field. We found that the staggered magnetic flux suppresses the long-range antiferromagnetic ordering. Peaks in the density of states are shifted by the magnetic flux. Magnetic-field-induced metal–insulator transitions can also be realized at both zero and non-zero temperature. An anisotropic 'd-wave' pseudogap is opened in the Fermi surface at half-filling. Thus most thermodynamical properties of the half-filled system should follow a power law at low temperatures. Finally, we introduced random fluctuations into the staggered flux and studied the effects of this on the density of states. We showed that the system developed a finite portion of Fermi surface with zero gap around four points $(\pm\pi/2, \pm\pi/2)$, whereas for the case of perfect staggered magnetic flux, there are only the four points on the Fermi surface with zero gap. This is very similar to what was observed for underdoped cuprates in angle-resolved photoemission spectra (ARPES) experiments. Thus the conductivity of the half-filled band would increase with the fluctuation. The bandwidth is a decreasing function of the fluctuation.

On the other hand, we found that the disturbed staggered field assumed in our model can actually be seen as a simulation of the spin fluctuations in the interacting system. This point is supported by many of the properties described above, which makes us believe that the features discussed in this paper are more physical for the weakly interacting system without the external field because the spin fluctuations have been reasonably well described. From this viewpoint, this can be seen as an improved method, which goes beyond the conventional mean-field approach for the interacting system.

Acknowledgments

The authors are grateful to Professor H Q Lin for stimulating discussions. We also wish to acknowledge the support of the Chinese Natural Science Foundation.

Appendix

In this appendix, we would like to show that the staggered flux pattern is the most energetically favourable state, as compared to the other patterns with non-zero flux. In fact, the staggered flux state represents an energy minimum with respect to the states with small deviations from it. To address this question, let us consider other configurations of flux, which have broken the translational symmetry and have small deviations from the staggered one. We treat these deviations as perturbations and calculate the ground-state energy. For simplicity, we consider the case of small U $(U < U_c)$ so that the long-range order is absent. For physical reasons, let us discuss the case shown in figure 5(a), where a symmetric deviation is imposed. With $\phi_1 = \phi - \delta\phi$ and $\phi_2 = \phi + \delta\phi$, the reduced Hamiltonian can be given by

$$H = -t\sum_k (2\mathrm{e}^{-\mathrm{i}\delta_1}(\cos k_x)B_k^+A_k + (\mathrm{e}^{\mathrm{i}\delta_3+\mathrm{i}k_y} + \mathrm{e}^{\mathrm{i}\delta_2-\mathrm{i}k_y})C_k^+A_k + (\mathrm{e}^{-\mathrm{i}\delta_3+\mathrm{i}k_y} + \mathrm{e}^{-\mathrm{i}\delta_2-\mathrm{i}k_y})D_k^+B_k + 2\mathrm{e}^{\mathrm{i}\delta_1}(\cos k_x)D_k^+C_k + \mathrm{h.c.}) \tag{A.1}$$

where A_k, B_k, C_k, D_k refer to annihilation operators for A, B, C, D sublattices respectively and the δ_i $(i = 1, 2, 3)$ are defined by

$$\delta_1 = \frac{\pi}{2}\phi/\phi_0 \qquad \delta_2 = \delta_1 - \frac{\pi\,\delta\phi}{\phi_0} = \delta_1 - y \qquad \delta_3 = \delta_1 + y \tag{A.2}$$

with $y = \pi\,\delta\phi/\phi_0$ being a small quantity. The energy spectrum reads

$$\epsilon_k = \pm\left\{\frac{1}{2}\{(2|a|^2 + |b|^2 + |c|^2) \pm [(2|a|^2 + |b|^2 + |c|^2)^2 - 4(|a|^4 + |b|^2|c|^2 - 2\mathrm{Re}(a^2bc))]^{1/2}\}\right\}^{1/2} \tag{A.3}$$

where

$$a = 2t\mathrm{e}^{\mathrm{i}\delta_1}\cos k_x \qquad b = 2t\mathrm{e}^{\mathrm{i}\delta_1}\cos(k_y + y) \qquad c = 2t\mathrm{e}^{\mathrm{i}\delta_1}\cos(k_y - y). \tag{A.4}$$

The ground state of the system is the state with the two lower sub-bands filled and the two higher sub-bands completely empty. After some tedious calculations, the ground-state energy of the system is as follows:

$$E = E_0 + \eta y^2 \tag{A.5}$$

where E_0 is the ground-state energy for the perfect staggered flux state and

$$\eta = \frac{2t^2\sin^2(\delta/2)}{|\cos(\delta/2)|}\sum_k \frac{|\cos k_y|}{|\cos k_x|}\left(\frac{1}{\sqrt{|a|^2+|d|^2-2|a||d||\cos(\delta/2)|}} - \frac{1}{\sqrt{|a|^2+|d|^2+2|a||d||\cos(\delta/2)|}}\right) > 0 \tag{A.6}$$

with $d = 2te^{i\delta_1}\cos k_y$. Obviously, the energy for the staggered flux state is the minimum one among those of these slightly deviating states, so the staggered pattern is actually the most energetically favourable one of all the fluctuating patterns with non-zero flux. This is physically because the staggered state preserves the translational symmetry and has an equivalent zero flux for any magnetic unit cell of the lattice [20].

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